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2016

MCM/ICM

Summary Sheet

Finding The Optimal Investment Strategy

Based on the MPT and Q-value-based Method

In order to improve the educational performance of the undergraduates in the United States, we establish a mathematical model based on the Markowitz Portfolio Theory and adopt the Q-value-based method to give the 1 to N optimized and prioritized candidate list of the colleges.

We screen out nine evaluation indicators with the method of correlation analysis and clustering analysis, such as completion rate of the students, proportion of students receiving the federal student loan and so on.

The most significant factor we focus on is the return on the charitable investment (ROI), which has the positive relationship with students' burden level and the effectiveness of the donation. More specifically, students' burden level is measured by the tuition fee and percentage of students who receive a Pell Grant or a federal student loan, while the effectiveness of the donation is related to the graduation rate, future income and tuition fee. Moreover, ROI follows the principle of diminishing marginal utility.

With the definition of ROI and the Markowitz Portfolio model, we can calculate the Q-value of each college. Then, we use the Q-value-based method to make the optimal charitable investment strategy. We add the memory factor to the ROI calculation formula and obtain the variation trend of investment in the next few years to get the time duration. What's more, we test our model by sensitivity analysis. In the test, we change the minimum investment amount to judge whether the strategy for the investment changes a lot.

In the end, we extend our model by analyzing the impact of the disciplines distribution on our result. Taking racial justice into consideration, we increase the priority of the schools which mainly serve for black or native people.

Keywords: **Markowitz Portfolio Theory, Q-value-based method, ROI, Justice**

Contents

1. Introduction：

1.1 Overview

As the Lumina Foundation insists, for individual Americans, the consequences of not completing postsecondary education are increasingly dire. Nowadays, more and more large charities attach great importance to the high-level education.

We try the best to develop the appropriate model and give the optimal investment strategy for the Goodgrant Foundation. The Goodgrant Foundation is a charitable organization that wants to help improve college students' educational performance in the United States. In this report, we will detail a series of generic mathematical models that choose the group of school of the most investment value according to the database. As well as the optimal charitable investment strategy for the Goodgrant Foundation, we give the definition of the return on investment(ROI) as the decision-making basis for other charity organization.

The Gates Foundation[1] specifically focuses on increasing opportunities for low income students and students from diverse racial and cultural backgrounds. Differently, the Lumina Foundation[2] is working to improve the number of highly educated people. These strategies are made by the large grant organizations based on the analysis of college students' current situation and do have obtained certain achievements.

However, without copying these policy of other charities, we try to find and establish the strategy from the investor's perspective. Contacting the economic principle and utilizing the economics model, our team gives the strategy to make the highest ROI by donating money to the groups of schools which worth the investment value, instead of these schools which need the investment.

1.2 Our work

The attached file provides various institution data. We transform the complicated and unordered information into readable and exploitable form by computer program of data processing. According to the model demand, we remove duplicate information, correct the obvious error during the period of data cleaning. We use the cluster analysis and the hierarchical analysis method to get the choice of major factors which greatly influence the charitable investment decision. We build the ROI model, give the definition of related variables and use the Markowitz Portfolio Theory to get the optimal solution. In order to get the mean-variance efficient portfolios and the capital market line, we separate the investment into two parts, one is the scholarship for students and the another one is for the improvement of school infrastructure. Take the tangent portfolio as the choice for the year investment. We consider the diminishing

marginal utility and use the Q-value-based method[6] to make the model close to the reality. The marginal utility is of convex function which shows reduced susceptibility to the same incentives. This effect makes us do the diversified choices to get the highest ROI. Moreover, the attenuation utility shows the change of situation of the college and we use the phenomenon to predict the time duration to have significant positive effect on student educational performance.

1.3 How the funding works

Three kinds of influence exist:

- Embodied in scholarship for outstanding students, motivating the students to have better academic performance
- Embodied in subsidies for needy students who require financial support, letting more students finish the college study
- Embodied in donations for improving the studying condition, such as setting up the database and increasing the number of Multi-media classrooms.

2. Nomenclature and assumptions

2.1Nomenclature

The list of nomenclature used in this report

Abbreviation	Description
ROI^k	Return on investment in k^{th} year
S^k	Sum of total income in k^{th} year
P^k	total amount of the donation in k^{th} year
Θ_i^k	Income of the school i in k^{th} year
λ_i^k	Return on venture investment of the school i in k^{th} year
λ_f^k	Return on risk-free investment in k^{th} year
$\mathrm{eff_i^k}$	Level of effectiveness for the investment of the school i in k^{th} year

$\mathrm{bur}_{\mathrm{i}}^{\mathrm{k}}$	Level of burden of the school i in k^{th} year
$UGDS_i^{\mathrm{k}}$	Total amount of the students
$\mathrm{RET\_FT}_{\mathrm{i}}^{\mathrm{k}}$	Retention rate of the students
$\mathrm{Gradrate}_{\mathrm{i}}^{\mathrm{k}}$	Completion rate of the students
$\mathrm{earn10}_{\mathrm{i}}^{\mathrm{k}}$	Capability to make profit of the students
$\mathrm{AVERAGENPT4}_{\mathrm{i}}^{\mathrm{k}}$	Average tuition of the college i in k^{th} year
$\mathrm{PCTFLOAN}_i^{\mathrm{k}}$	Percent of students receiving a federal student loan
$\mathrm{PCTPELL}_{\mathrm{i}}^{\mathrm{k}}$	Percentage of students receiving a Pell Grant

2.2 Assumptions

- The amount of the charitable investment for each college has the minimum limit. The amount of the funding is an integer multiple of USD 50,000.
- Removing the colleges with plentiful missing data from the choice list will not affect the accuracy of the final result.
- There is no emergency which suddenly changes the condition of the college educational performance.
- The strategy of great charitable organization is appropriate and of reference value
- The data is accurate and shows the real condition of the college.
- The provided data is sufficient to judge the educational performance.
- The investment to the college follows the principle of diminishing marginal utility.
- We only consider how to make the investment strategy. The colleges are able to use the funding appropriately and efficiently.
- No money remains to the next period for future use.

3 Data processing and selecting indicator

3.1 Data preprocessing

The attached data file and the online EPIDS Database provide us with data of 7804 colleges and universities in the United States. The data includes various aspects of the colleges' information. We choose the 2977 candidate schools given in the file as the choice for our charitable investment. After determining the scope of candidate schools, we firmly believe that we should focus on the schools with complete data and good fame. We delete these closed schools and the colleges whose data is completely insufficient from the list of candidates, in order to get the final list. The rest 2600 colleges, with ample information, make up our choice scope of the investment decision. Noticing that some of the data is missing, we adapt the interpolation method to fill the data according to the correlation.

3.2 Clustering analysis

We get a total of 1721 kinds of elements covering all aspects of data for each college. It is meaningless to take all of the aspects in to consideration. We use the R language to do the clustering[3] analysis of the data categories. The factors mirror the condition of the colleges from six major aspects:

- school basic information (tuition, scale, courses);
- quality of the freshmen;
- quality of the graduates;
- satisfaction of the students;
- burden if the students;
- social evaluation.

Among the six major kinds of the data, we select 12 representative data to further analyze whether the college is worth the investment.

Average tuition	UG25abv	C150_4_POOLED_SUPP
PCTPELL [7]	md_earn_wne_p10	gt_25k_p6
RET_FT4	GRAD_DEBT_MDN10YR_SUPP	SAT_AVG_ALL
PCTFLOAN	RPY_3YR_RT_SUPP	UGDS

3.3 Major index selecting

First of all, we study the factors which Gates Foundation and other large charity organization attach great importance to in their donating strategy. We calculate the

correlation of the 14 elements and the 2015 large-scale charity funding scheme.We can see that the large charity organization focus on RET_FT4, gt_25k_p6, SAT_AVG_ALL, RPY_3YR_RT_SUPP, md_earn_wne_p10, C150_4_POOLED_SUP. Three of the six elements, md_earn_wne_p10, RPY_3YR_RT_SUPP, gt_25k_p6 all show the same aspect of the graduates. We select the md_earn_wne_p10 as the representative to evaluate the capacity of making profit. There remain four key elements: C150_4_POOLED_SUP, RET_FT4, SAT_AVG_ALL, md_earn_wne_p10, RET_FT4.

Combining the document literature and our own experience, we recognized that the student's need for charitable fund also depend on the PCTFLOAN and PCTPELL. Both of the two rates reflect the burden of students in the college. What's more, the amount of the Average tuition determines the extent of help fixed number of subsidies give to each student, this is also the key factors affecting the efficiency of the investment.

In conclusion, we get seven aspects of the reference data that play an important role in measuring the efficiency of the investment.

3.4 Indicators for the model improvement

In the model, we also take the elite in to consideration. The ROI [9]have the trend to increase significant when donate the money to the high-level university. High-level and famous colleges require the students to process good academic performance. This aspect is measured by the SAT_AVG_ALL in the data.

To develop the model in the perspective of equity, we also take racial proportion into consideration. We try to avoid donating money to the colleges with the same major course, like the Business. The goal is to let the Goodgrant Foundation help and inspire more people, instead of concentrating in a certain crowd. We encourage students to choose different subject. The society needs all kinds of skills and talents.

4. Model 1: The Maximizing ROI model based on the Markowitz Portfolio Theory

4.1 How to build the model

Why do we need to follow the traditional philanthropic investment strategy? Without duplicating the strategy of large charities, we not only consider which college needs the fund but also transform the perspective to analyze how to distribute the

US100 million, in order to optimize the return on investment and achieve full use of the funds to afford maximizing social benefits. We start from the perspective of investor and change the task into making investment portfolio decisions. The aim is to maximize the rate of return under the same level of risk. We consider two forms of philanthropic investment, one for the personal students and one for the college construction. One the one hand, the former one is of high risk due to the indeterminacy. The student who receives the scholarship may drop out of school or abuse the money. On the another hand, if the student benefits from the scholarship, like being motivated and being able to complete the whole college study, the personal investment is of relatively high return. The second kind of investment for construction of the college, such as building database, expanding the library, can steadily benefit all of the students. The function is stable and it takes a long period to have significant positive influence. We think the former kind of investment is investing in risk assets. Similarly, the investment for college construction is kind of risk-free assets investment. So we change the question to analyze the portfolio in the background of the above explain.

Two parts of the considerations in the charitable investment, the return rate and the risk, exactly coincided with the hypothesis of Markowitz Portfolio Theory. So, we can use the Markowitz Portfolio Theory to help us find the optimal strategy. What' more, according to the discussion of the principle of diminishing marginal utility, we already know it is not a good choice to make the decision for US100 million at one time. We take the diminishing marginal utility of the one-school investment into consideration and adopt the Q-value-based method to establish dynamic programming model.[4] Separate the one-year investment into several stages and make the decision respectively. Then we get the yearly optimal investment strategy and use the same way to get the strategy for the next five years. In the last, by comparing the strategies of different years, we judge the time duration that the money should be provided to have a significant positive effect on students' performance.

4.2 Markowitz Portfolio Theory

What is MPT[5]? The concept behind MPT is that the assets in an investment portfolio should not be chose merely individually, each on its own character. It is important for us to consider how each asset might change in price relative to how every other asset in the portfolio might change in price.

When we make an investment decision, it is a tradeoff between risk and expected return. An efficient portfolio is said to be having a combination of at least two stocks considering the minimum variance portfolio. MPT gives us the way to select a portfolio with the highest possible expected return under a given amount of risk.

We decide to make a choice from N assets $\boldsymbol{Q_i(i = 1, 2 \ldots, N)}$.

We have the expected return for each asset as r_i and the correlation coefficient ρ_{ij} between the returns on asset $\mathbf{Q_i}$ and $\boldsymbol{Q_j}$, then we get the covariance matrix $\boldsymbol{\Sigma}$.

We definite the matrix $\boldsymbol{r_m = [r_1, r_2 \ldots, r_N],\ \iota = [1, 1, 1 \ldots, 1]}$

If the amount of the investment is fixed, then the question to find the portfolio with the highest rate is equal to find the weight vector $\boldsymbol{\Omega}$ which shows the distribution of the investment. To find the portfolio with highest possible expected return under a given amount of risk is equal to find the lowest risk under a given expected return $\mathbf{r_p}$.

$$\boldsymbol{min_z} \quad \frac{1}{2}\boldsymbol{\Omega^\tau \Sigma \Omega} \tag{1}$$

$$\boldsymbol{s.t.} \begin{cases} \boldsymbol{\Omega^\tau r_m = r_p} & (2) \\ \boldsymbol{\Omega^\tau \iota = 1} & (3) \end{cases}$$

With arithmetical operation, we can get the Ω, which is called the mean-variance frontier portfolio

We definite $\boldsymbol{a = \iota^\tau \Sigma^{-1} r_m}$, $\boldsymbol{b = r_m{}^\tau \Sigma^{-1} r_m}$, $\boldsymbol{c = \iota^\tau \Sigma^{-1} \iota}$, $\boldsymbol{d = bc - a^2}$, $\boldsymbol{\lambda_r = d^{-1}(cr_p - a)}$, $\boldsymbol{\lambda_1 = d^{-1}(b - ar_p)}$ to simplify the expression of $\boldsymbol{\Omega}$.

$$\boldsymbol{\Omega = \lambda_r \Sigma^{-1} r_m + \lambda_1 \Sigma^{-1} \iota} \tag{4}$$

For each $\boldsymbol{r_p}$, we can find a $\boldsymbol{\Omega}$. Give the $\boldsymbol{r_p}$ with different value, we can get the mean-variance frontier.

We try to find the relationship of the frontier portfolio's variance $\boldsymbol{\sigma_p}$. To make it easy to understand, we define $\boldsymbol{z_0 = d^{-1}(b \Sigma^{-1} \iota - a \Sigma^{-1} r_m)}$, $\boldsymbol{z_1 = z_0 + d^{-1}(c \Sigma^{-1} r_m - a \Sigma^{-1} \iota)}$.

Then we can get the expression of the $\boldsymbol{\sigma_p}$.

$$\boldsymbol{\sigma_p{}^2 = [z_0 + r_p(z_1 - z_0)]^\tau \Sigma [z_0 + r_p(z_1 - z_0)]} \tag{5}$$

Every possible combination of the mean-variance frontier portfolio can be plotted in risk-expected return space. Take the risk-free invest into consideration, the return rate of the risk-free invest is r_f.Then we can get the tangent portfolio. And this portfolio is the one we choose for the optimal strategy.

4.3 Q-value-based method theory

Q-value-based method[10] is a kind of fair model for quota allocation which is presented by an American mathematician in 1880. The main algorithm of the method is to use the Q-value which is calculated by us to allocate the additional quota to the candidate who has the highest Q-value. After one allocation, we recalculate the Q-value and repeat the process until all quotas has been allocated. Through Q-value-based method, we can allocate all of our investment reasonably and fairly.

In this case, we can get the Q-value from the Markowitz Portfolio Theory.

According the basic economy theory, the efficiency of the investment will decrease when the number of investment increases. Here we use infinitesimal dividing modeling method to divide our contribution into many small parts, so that when we invest the small part to a target school, the Q-value of the school can be assumed to be constant. Then we allocate the small parts to the schools with Q-value method and then we can achieve reasonable allocation.

4.4 First-year model implementation and the result

Seeking for the one-year optimal investment strategy, we consider the ROI as the return rate resulting from the investment. The aim is to evaluate the efficiency of the charitable investment. And in the model, we define the ROI as the measure of rates of return on money donated to the group of school. It follows that:

$$\boldsymbol{ROI^1 = \frac{S^1}{P^1}} \quad (6)$$

The total amount of the money is US100 million and the income of the portfolio is the sum of the income resulting from each single invests.

$$\boldsymbol{S^1 = \sum_{i=1}^{2600} \Theta_i^1} \quad (7)$$

Then for each school $\boldsymbol{i}$, if the amount of the investment is $\boldsymbol{\varphi_i^1}$,and the proportion of the risk-free investment is a, then the expression of the $\boldsymbol{\Theta_i^1}$ follows :

$$\boldsymbol{\Theta_i^1 = (1-a)\,\varphi_i^1\lambda_i^1 + a\,\varphi_i^1\lambda_f^1} \quad (8)$$

Here, as we have made the explain fin the assumption, the rate $\boldsymbol{a}$ is not the main point of the discussion. The value of the $\boldsymbol{\Theta_i^1}$ equals to $\boldsymbol{\varphi_i^1\lambda_i^1}$

And how is the $\boldsymbol{\lambda_i^1}$ defined? Our team firmly believes that the ROI of the charitable investment is closely related to the student's burden level and the effectiveness of the donation.

$$\boldsymbol{\lambda_i^1 = bur_i^1 * eff_i^1} \quad (9)$$

bur_i^1 is measured by the students' economic burden in the college, with the rate showed an inverse relationship to $\boldsymbol{PCTPELL_i^1}$. Likewise, the increase of the $\boldsymbol{PCTFLOAN_i^1}$ reduce the value of $\boldsymbol{bur_i^1}$. The $\boldsymbol{AVERAGENPT4_i^1}$ shows the level of the tuition and is the main reason of the students' economic burden. Increasing the number of the investment has a positive effect to relieve the students' economic pressure. However, the number of students $\boldsymbol{UGDS_i^1}$ shows the proportion of the students who can get the money. So, if the current investment amount is $\boldsymbol{\varphi_{ic}^1}$,

$$\boldsymbol{bur_i^1 = \left(1-PCTPELL_i^1\right)\left(1-PCTFLOAN_i^1\right) * AVERAGENPT4_i^1 - \frac{\varphi_{ic}^1}{UGDS_i^1}} \quad (10)$$

Then the effectiveness of the investment is measured by the effective degree of

unit investment. It has a positive relationship with $\boldsymbol{Gradrate_i^1}$,$\boldsymbol{RET\_FT_i^1}$ and the scale of the school. The ability of the students to make profit shows the quality of the college graduates which is the significant factor of the $\boldsymbol{eff_i^1}$And the higher the tuition is, the more the money student need to pay for the college, the less the effectiveness is. So,

$$\boldsymbol{eff_i^1 = \frac{earn10_i^k}{AVERAGENPT4_i^1} * (UGDS_i^1 * Gradrate_i^1 * RET\_FT_i^1)} \qquad (11)$$

We can get all the numerical value of the parameters from the data given in the attached file. We calculate the numeric value of the $\boldsymbol{\Theta_i^1}$ for the 2600 schools.
And we get the matrix $\boldsymbol{r_m}$ defined before in the passage. $\boldsymbol{r_m = [\Theta_1^1, \Theta_2^1 \ldots, \Theta_{2600}^1]}$
By processing data from 2007 to 2013, we can figure out the return rate of investment for different colleges and get the covariance matrix $\boldsymbol{\Sigma}$[8].

In order to get the optimal investment strategy and maximize the efficiency, we adopt the Q-value-based method to separate the investment into 20 stages. At each stage, we make the decision of the allocation of US 500,000. Then use the Markowitz portfolio model to get the mean-variance frontier, and further get the tangent portfolio.

According to the assumption, the minimum number of the investment is US 50,000. We choose the college due to ranking the weight of the vector $\boldsymbol{\Omega}$.

Table 1. First stage of the investment

Ranking	University	Θ_i^1
1	Texas A & M University-College Station	16.0704
2	University of California-Berkeley	15.8561
3	University of Michigan-Ann Arbor	15.8299
4	University of California-Los Angeles	15.8021
5	University of Florida	15.7845
6	The University of Texas at Austin	15.7393
7	Texas A & M University-College Station	15.8949
8	University of Illinois at Urbana-Champaign	15.6311
9	University of Maryland-College Park	15.6033
10	University of Washington-Seattle Campus	15.6447

Due to the Q-value-based method, we adjust the expected value of the Θ_i^1 based on the number of the investment the college already have this year. And then we start the second stages. Following the same steps of the first stages, use the Markowitz portfolio model again, we can get the choice of the second stage.

Table 2. Second stage of the investment

Ranking	University	$\boldsymbol{\Theta_i^1}$
1	Ohio State University-Main Campus	15.5966
2	Texas A & M University-College Station	15.8117
3	Brigham Young University-Provo	15.5272

4	University of Wisconsin-Madison	15.5243
5	Texas A & M University-College Station	15.7486
6	Harvard University	15.351
7	Texas A & M University-College Station	15.6976
8	Georgia Institute of Technology-Main Campus	15.3861
9	University of California-Berkeley	15.6404
10	Michigan State University	15.4814

Then the third stage of choice .

Table 3. Third stage of the investment

Ranking	**University**	$\boldsymbol{\Theta_i^1}$
1	University of Georgia	15.4194
2	University of Virginia-Main Campus	15.353
3	University of Michigan-Ann Arbor	15.6143
4	Texas A & M University-College Station	15.6545
5	University of California-Los Angeles	15.6001
6	University of Florida	15.5867
7	University of California-San Diego	15.3493
8	Texas A & M University-College Station	15.6171
9	University of North Carolina at Chapel Hill	15.3191
10	Indiana University-Bloomington	15.3729

Again and again, after the twentieth stage of choice, we get the final strategy for one year investment. The strategy is showed in the following sheet.

Table 4. The optimal investment strategy for the first year $\boldsymbol{ROI^1 = 15.07284\%}$

Rank	**University**	**Investment amount(USD)**
1	Texas A & M University-College Station	15500000
2	University of California-Berkeley	8000000
3	University of California-Los Angeles	7000000
4	University of Michigan-Ann Arbor	7000000
5	University of Florida	6500000
6	The University of Texas at Austin	5500000
7	University of Washington-Seattle Campus	4500000
8	University of Illinois at Urbana-Champaign	3500000
9	University of Maryland-College Park	3500000
10	Ohio State University-Main Campus	3000000
11	Michigan State University	2500000
12	Brigham Young University-Provo	2500000
13	University of Wisconsin-Madison	2500000
14	University of California-Irvine	1500000
15	University of California-San Diego	1500000

16	De Anza College	1500000
17	University of Central Florida	1500000
18	Georgia Institute of Technology-Main Campus	1500000
19	University of Georgia	1500000
20	Indiana University-Bloomington	1500000
21	Pennsylvania State University-Main Campus	1500000
22	University of California-Davis	1000000
23	Harvard University	1000000
24	Rutgers University-New Brunswick	1000000
25	University of North Carolina at Chapel Hill	1000000
26	Virginia Polytechnic Institute and State University	1000000
27	University of Virginia-Main Campus	1000000
28	Purdue University-Main Campus	1000000
29	Arizona State University-Tempe	500000
30	California Polytechnic State University-San Luis Obispo	500000
31	University of California-Santa Barbara	500000
32	San Diego State University	500000
33	University of Southern California	500000
34	University of Connecticut	500000
35	University of Delaware	500000
36	Florida State University	500000
37	Louisiana State University and Agricultural & Mechanical College	500000
38	University of Minnesota-Twin Cities	500000
39	Princeton University	500000
40	Columbia University in the City of New York	500000
41	Cornell University	500000
42	New York University	500000
43	North Carolina State University at Raleigh	500000
44	University of Pennsylvania	500000
45	Clemson University	500000
46	James Madison University	500000
47	Stanford University	500000

4.5 Prediction and analysis of the investment for the next four years

Our contributions need a kind of continuity of certain period of time, while repeated investment will decrease the efficiency of investment. Therefore we need to introduce a factor to remember the investment the school has achieved which will decrease the ROI after a yearly contribution. When we take this factor into account and adopt the same allocation strategy as we used before, we will get the investment

of the next five years.

strategy for the second year:

Table 5. The optimal investment strategy for the second year $ROI^2 = 15.0057\%$

Rank	University	Investment amount(USD)
1	Texas A & M University-College Station	15500000
2	University of California-Berkeley	7500000
3	University of California-Los Angeles	7000000
4	University of Michigan-Ann Arbor	7000000
5	University of Florida	6500000
6	The University of Texas at Austin	5500000
7	University of Washington-Seattle Campus	4500000
8	University of Illinois at Urbana-Champaign	3500000
9	University of Maryland-College Park	3500000
10	Ohio State University-Main Campus	3000000
11	Michigan State University	2500000
12	Brigham Young University-Provo	2500000
13	University of Wisconsin-Madison	2500000
14	University of California-Irvine	1500000
15	De Anza College	1500000
16	University of Central Florida	1500000
17	Georgia Institute of Technology-Main Campus	1500000
18	University of Georgia	1500000
19	Indiana University-Bloomington	1500000
20	Pennsylvania State University-Main Campus	1500000
21	University of Alaska Anchorage	1000000
22	University of California-Davis	1000000
23	University of California-San Diego	1000000
24	Harvard University	1000000
25	Rutgers University-New Brunswick	1000000
26	University of North Carolina at Chapel Hill	1000000
27	Virginia Polytechnic Institute and State University	1000000
28	University of Virginia-Main Campus	1000000
29	Purdue University-Main Campus	1000000
30	Arizona State University-Tempe	500000
31	California Polytechnic State University-San Luis Obispo	500000
32	University of California-Santa Barbara	500000
33	San Diego State University	500000
34	University of Southern California	500000

35	University of Connecticut	500000
36	University of Delaware	500000
37	Florida State University	500000
38	Louisiana State University and Agricultural & Mechanical College	500000
39	University of Minnesota-Twin Cities	500000
40	Princeton University	500000
41	Columbia University in the City of New York	500000
42	Cornell University	500000
43	New York University	500000
44	North Carolina State University at Raleigh	500000
45	University of Pennsylvania	500000
46	Clemson University	500000
47	James Madison University	500000
48	Stanford University	500000

strategy for the third year

Table 6. The optimal investment strategy for the third year

$\boldsymbol{ROI^3 = 14.95773\%}$

Rank	University	Investment amount(USD)
1	Texas A & M University-College Station	13000000
2	University of California-Berkeley	8000000
3	University of California-Los Angeles	7000000
4	University of Florida	6500000
5	University of Michigan-Ann Arbor	6500000
6	The University of Texas at Austin	5500000
7	University of Washington-Seattle Campus	4500000
8	University of Illinois at Urbana-Champaign	4000000
9	University of Maryland-College Park	3500000
10	Ohio State University-Main Campus	3500000
11	Brigham Young University-Provo	3000000
12	University of Wisconsin-Madison	3000000
13	Michigan State University	2500000
14	De Anza College	2000000
15	University of Georgia	2000000
16	Indiana University-Bloomington	2000000
17	Pennsylvania State University-Main Campus	2000000
18	University of Alaska Anchorage	1500000
19	University of California-Davis	1500000
20	University of California-San Diego	1500000
21	University of Central Florida	1500000
22	Georgia Institute of Technology-Main Campus	1500000

23	Harvard University	1500000
24	University of North Carolina at Chapel Hill	1500000
25	University of Virginia-Main Campus	1500000
26	Purdue University-Main Campus	1500000
27	Florida State University	1000000
28	University of Minnesota-Twin Cities	1000000
29	Rutgers University-New Brunswick	1000000
30	Virginia Polytechnic Institute and State University	1000000
31	California State University-Fullerton	500000
32	University of Colorado Boulder	500000
33	Yale University	500000
34	Georgetown University	500000
35	Iowa State University	500000
36	Boston University	500000
37	CUNY Bernard M Baruch College	500000
38	Vanderbilt University	500000

strategy for the fourth year

Table 7. The optimal investment strategy for the fourth year $\boldsymbol{ROI^4 = 14.91373\%}$

Rank	University	Investment amount(USD)
1	Texas A & M University-College Station	11000000
2	University of California-Berkeley	7500000
3	University of California-Los Angeles	7000000
4	University of Florida	6500000
5	University of Michigan-Ann Arbor	6500000
6	The University of Texas at Austin	5500000
7	University of Washington-Seattle Campus	5000000
8	University of Illinois at Urbana-Champaign	4000000
9	University of Maryland-College Park	4000000
10	Ohio State University-Main Campus	3500000
11	Brigham Young University-Provo	3500000
12	Michigan State University	3000000
13	University of Wisconsin-Madison	3000000
14	De Anza College	2500000
15	University of Georgia	2500000
16	University of California-Irvine	2000000
17	University of California-San Diego	2000000
18	University of Central Florida	2000000
19	Georgia Institute of Technology-Main Campus	2000000
20	Indiana University-Bloomington	2000000

21	Pennsylvania State University-Main Campus	2000000
22	University of Virginia-Main Campus	2000000
23	University of California-Davis	1500000
24	Rutgers University-New Brunswick	1500000
25	Virginia Polytechnic Institute and State University	1500000
26	University of Southern California	1000000
27	Cornell University	1000000
28	University of Pennsylvania	1000000
29	University of Arizona	500000
30	California State University-Long Beach	500000
31	University of South Florida-Main Campus	500000
32	Northwestern University	500000
33	University of Notre Dame	500000
34	University of Iowa	500000
35	University of Missouri-Columbia	500000

strategy for the fifth year

Table 8. The optimal investment strategy for the fifth year $\boldsymbol{ROI^4 = 14.86032\%}$

Rank	University	Investment amount(USD)
1	Texas A & M University-College Station	9000000
2	University of California-Berkeley	7500000
3	University of California-Los Angeles	6500000
4	University of Florida	6000000
5	University of Michigan-Ann Arbor	6000000
6	The University of Texas at Austin	5000000
7	University of Washington-Seattle Campus	5000000
8	University of Illinois at Urbana-Champaign	4000000
9	University of Maryland-College Park	4000000
10	Ohio State University-Main Campus	3500000
11	Brigham Young University-Provo	3500000
12	University of Georgia	3000000
13	Michigan State University	3000000
14	University of Wisconsin-Madison	3000000
15	University of California-Irvine	2500000
16	Georgia Institute of Technology-Main Campus	2500000
17	Indiana University-Bloomington	2500000
18	University of California-Davis	2000000
19	University of California-San Diego	2000000
20	University of Central Florida	2000000

21	Harvard University	2000000
22	University of North Carolina at Chapel Hill	2000000
23	Pennsylvania State University-Main Campus	2000000
24	Purdue University-Main Campus	2000000
25	University of Alaska Anchorage	1500000
26	Florida State University	1500000
27	Rutgers University-New Brunswick	1500000
28	Virginia Polytechnic Institute and State University	1500000
29	De Anza College	1000000
30	University of Minnesota-Twin Cities	1000000
31	The University of Alabama	500000
32	University of South Carolina-Columbia	500000
33	George Mason University	500000

The following graph reveals the change of investment of the top 5 schools in the next five years.

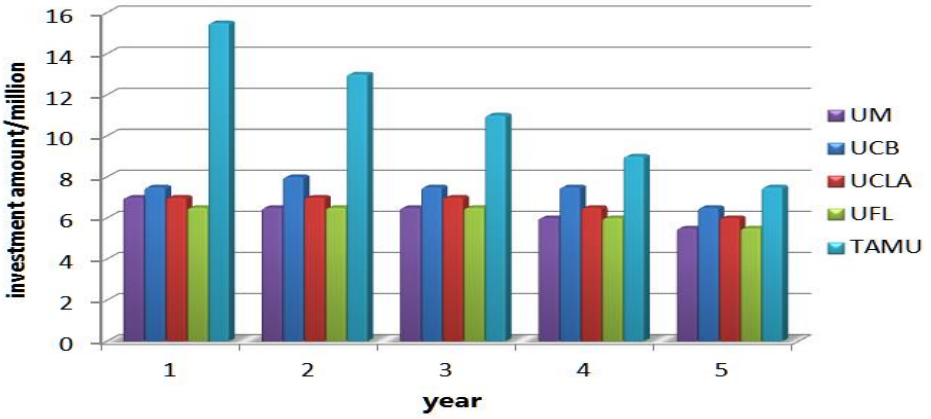

Figure 1. The change of investment of the top 5 schools in the next five years

We find that the investment will be evenly contributed as time flows. Our contribution will become more balanced. This result reflects our assumption that repeated investment will lead to low efficiency.

4.7 Time period analysis

We also make the investment change in the next 9 years.

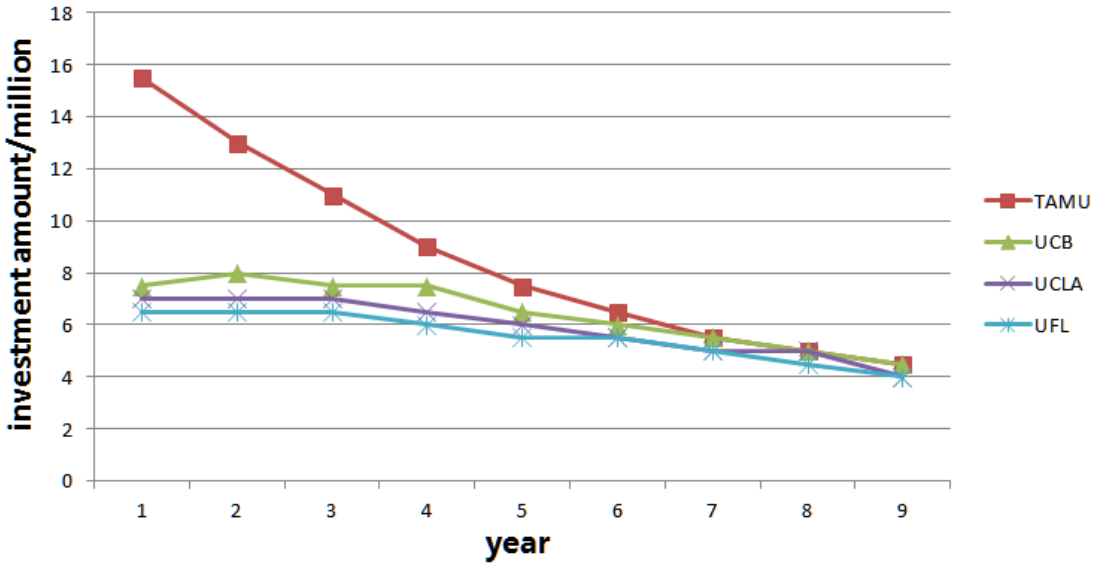

Figure 2. The investment to the top 4 schools in the following 5 years

We conclude that about seven years, our investment will have evident change and our plan will improve the balance of education resources. This change shows the significant positive effect on the college students' educational performance. And the time period that her money should be provided is Seven years.

5. Model 2: the ROI model based on the perspective of justification

When we calculate the ROI of every school, we ignore the distribution of the discipline. Meanwhile, we have not taken whether the school is predominantly serving for the black or native people. To make our contribution more fairly, we need to take these indexes into account.

Firstly, we analyze the correlation between the contributions and the discipline distribution. Here we classify the disciplines into Humanities, Science and Engineering, Business and Others. We use the proportion of graduates of every subject to represent the discipline distribution of every school.

We make the contributions vector: ***contribution,*** proportion of humanity vector: ***humanity***, proportion of science and engineering: ***business***, vector proportion of business vector proportion of others vector: ***others***. By calculating the correlation between ***contribution*** and ***liberal***, ***science*** and ***others***, we can judge whether the distribution of disciplines will affect our contributions.

The formula of the correlation coefficient is as follows:

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}} = \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{\sqrt{(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2)(\sum_{i=1}^{n} y_i^2 - n\bar{y}^2)}} \quad (12)$$

Table 9. Correlation between contribution and disciplines

Correlation	**humanity**	**science**	**business**	**others**
contribution	-0.156	0.134	-0.095	0.045

We find that contributions and discipline distribution only have a weak correlation, so that our contribution is nearly fair to every discipline.We find that we fail to give our contributions to the school which is predominantly serving for the back and native people. To make our contribution fairer for races or regions, we can change our former strategy to calculate Q-value. We can raise the priority for these schools by introducing a coefficient: **bn**. If the school is mainly serving for black or native people, the assignment can be larger than 1. Otherwise, the assignment of **bn** is 1.

The different assignment of **bn** will have different effect on our result. The following graph reveals the relationship between the assignment of **bn** and the money we contribute to the schools which are mainly serving for the black and the native.

In the following figure, we use fair investment to represent the money we contribute to the schools which are mainly serving for the black people and native people.

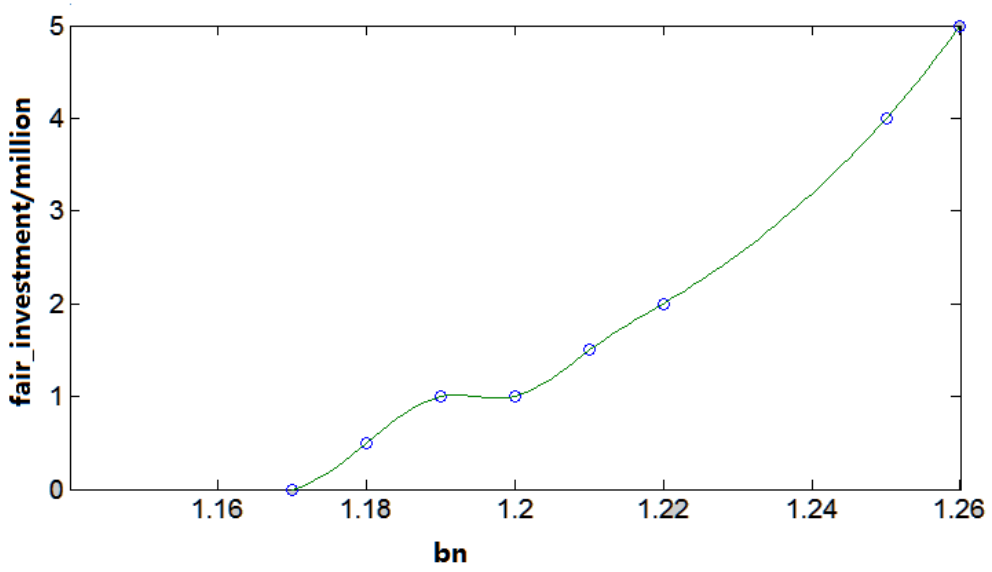

Figure 3. The relationship between amount of money we contribute to schools mainly serving for the black or the native and **bn**

We conclude from the figure that we need to set the coefficient larger than 1.17 or we will make no change. Meanwhile, we find that when the coefficient is smaller than 1.27, there is only one school serving for the native (UAA) can be contributed. When the coefficient keeps growing, most of the money will go to UAA. It reflects that most schools which is serving for the black or the native have too low ROI. As a result, the assignment of 1.2 is a reasonable choice. Based on the choice of the **bn**, another optimal investment strategy is available with the same steps in the Model 1.

6. Sensitivity test

In previous section, we assume that the minimum investment amount is 1 million dollars. This assumption of this value is based on experience. Therefore, we need to change the value of minimum investment amount to see whether it is rational. To save space, we only listed the top 5 universities.

Table 10. Rank of the investment on universities (min_investment=250,000)

Rank	University	Investment amount	ROI
1	***TAMU***	**16250000**	**15.0889**
2	***UCB***	**8250000**	**15.0822**
3	***UCLA***	**7250000**	**15.0788**
4	***UM***	**7250000**	**15.0887**
5	***UFL***	**6750000**	**15.0848**

Table 11. Rank of the investment on universities (min_investment=500,000)

Rank	University	Investment amount	ROI
1	***TAMU***	**15500000**	**15.109**
2	***UCB***	**8000000**	**15.0923**
3	***UCLA***	**7000000**	**15.0901**
4	***UM***	**7000000**	**15.1**
5	***UFL***	**6500000**	**15.0967**

Table 12. Rank of the investment on universities (min_investment=1,000,000)

Rank	University	Investment amount	ROI

1	***TAMU***	**15000000**	**15.1226**
2	***UCB***	**7000000**	**15.135**
3	***UCLA***	**6000000**	**15.1378**
4	***UFL***	**6000000**	**15.1215**
5	***UM***	**6000000**	**15.1483**

Table 13. Rank of the investment on universities (min_investment=1,250,000)

Rank	University	Investment amount	ROI
1	***TAMU***	**13750000**	**15.1578**
2	***UCB***	**6250000**	**15.1698**
3	***UCLA***	**6250000**	**15.1254**
4	***UM***	**6250000**	**15.1357**
5	***UFL***	**5000000**	**15.1756**

Table 14. Rank of the investment on universities (min_investment=2,000,000)

Rank	University	Investment amount	ROI
1	***TAMU***	**10000000**	**15.276**
2	***UCB***	**6000000**	**15.182**
3	***UCLA***	**4000000**	**15.2535**
4	***UFL***	**4000000**	**15.2381**
5	***UM***	**4000000**	**15.2649**

Table 15. Rank of the investment on universities (min_investment=2,500,000)

Rank	University	Investment amount	ROI
1	***TAMU***	**7500000**	**15.3712**
2	***UCB***	**5000000**	**15.235**
3	***UCLA***	**5000000**	**15.1915**
4	***UM***	**5000000**	**15.2024**
5	**UC Davis**	**2500000**	**14.882**

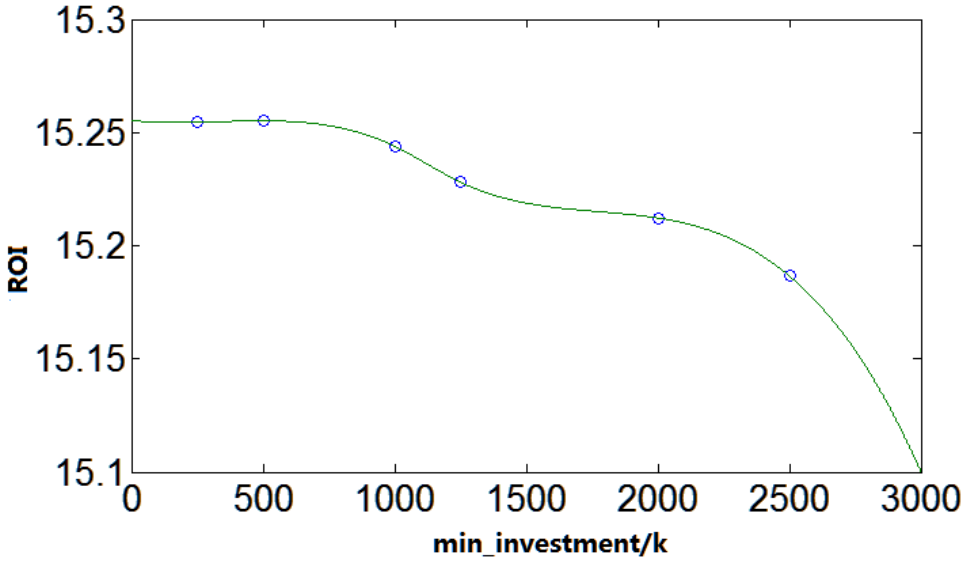

Figure 4. ROI curve of different minimum investment amount

By comparing the above tables, we find that the top 5 universities are almost identical, which means that our model is insensitive to the change of the minimum investment amount. However, the funds invested in the universities vary from year to year.

What's more, it can be seen from Figure 1 that selecting 1,000,000 dollars as the minimum investment amount not only has a high ROI, but also simplify the allocation of funds.

7. Final remarks

7.1 Strengths

- We make full use of the datum from the database provided by IPEDS, including the historical datum from 2007 to 2013. As a result, we do have a more comprehensive assessment of the schools. We try the best to include more potential effective indicators in the process of modeling in order to have a comprehensive strategy.

- We combine the charitable investment with the economic theory. View the task with the prospective from the investor. The model based on the MPT and Q-value-based method is effective in finding the optimal investment strategy.

- We make use of Q-value-based method to allocate the contribution reasonably. We combine Q-value method and Markowitz Mean-Variance Model with each other to avoid the weakness of Q-value based method that lacks criteria for evaluation and the weakness of MPT that ignores the influence of investment on the change of Q-value.

- We take some indexes like the distribution of courses of the college and the racial justice into consideration in order to ensure the justification of the strategy.

- We introduce the memory factor to adjust the Q-value according to the historical investment so our contribution will become more appropriate in the next five years.

7.2 Weakness:

- Some of the colleges lack part of the dat. We have to fill them by analyzing the relation with other datum. However, these hypothetical data may not reflect the real educational performance of these colleges.

- The formula of efficiency and burden is mainly based on our study of large amounts of documents and combined with our experimental hypothesis. Therefore, it may lack accuracy.

- Our model takes the risk-free investment into consideration. However, with insufficient relevant data, it is a challenge for us to calculate the exact return rate of the risk-free charitable investment.

7.3 Further model develop and improvement

Since the volatility of different colleges' ROI and the correlation between colleges are inconspicuous, we need to optimize our index of risk. We could take the uncertainty of the performance of the student who achieved our contribution into the risk analysis in the future. By using the data of the income distribution, we can analyze the uncertainty of the investment more reasonably.

Since we have little data about the college's reflection to the charitable investment, the influence to the colleges is based on the analysis of documents and our conjecture. In the future development of the model, we should observe the effects of the investment in order to better understand the college transforming process.

7.4 Conclusions

We establish the model to find the optimal charitable investment. Firstly, we handle the data from the attached fire and choose the major factors. Then we establish the model for highest ROI based on the MPT and Q-value-based method. In order to take the fairness in to considerable, we add the racial rate and colleges' category to the new model. We get the strategy for the Goodgrant Foundation in the coming five years according to some valuable kinds of data. Finally ,we know that the time period of the money should be provided to have significant good influence to the educational performance is 7 years.

8. References

[1]http://www.gatesfoundation.org/What-We-Do/US-Program/Washington-State/Education-Pathways
[2]https://www.luminafoundation.org/files/resources/2013-lumina-strategic-plan.pdf
[3] Xu R,Wunsch D. Survey of clustering algorithms[J]. IEEE Trans on Neural Networks, 2005, 16(3): 645-678.
[4] Wu H, Zeng Y, Yao H. Multi-period Markowitz's Mean-Variance Portfolio Selection with State-dependent Exit Probablity. Economic Modelling, 2014,36:69-78
[5] Harry Markowitz. Portfolio Selection[J]. The Journal of Finance, 1952,7:77-91
[6] Huntley I D, James D J G. Mathematical Modelling [M]. Oxford: Oxford University Press, 1990.
[7] http://www.simpletuition.com/pell-grant/
[8] https://collegescorecard.ed.gov
[9]Boyd J, Epanchinniell R, Siikamäki J. Conservation Planning: A Review of Return on Investment Analysis[J]. Review of Environmental Economics & Policy, 2015, 9.
[10] Yang Y M, Sha F. The Multi-Attributes Seat Apportionment Model with Time Series and Its Application[J]. Mathematics in Practice & Theory, 2012, 42(6):95-102.

A letter to the Chief Financial Officer

Dear Mr. Alpha Chiang:
As is universally acknowledged, a rational university investment strategy is critical for the improvement of educational performance of the undergraduates. In order to achieve the goal of the Goodgrant Foundation, we establish a mathematical model based on the Markowitz Portfolio Theory.

To begin with, the ROI of the charitable investment we proposed is proportional to school size, students' burden level and the effectiveness of the donation. More specifically, we use the number of students to represent school size. Students' burden level includes the tuition fee and percentage of students who receive a Pell Grant or a federal student loan, while the effectiveness of the donation is composed of graduation rate, future wage and tuition fee. Moreover, ROI will decrease with the increasement of the investment. The reduction extent is proportional to amount of contributions and is inversely proportional to the number of smart students, while the number of smart students is proportional to the graduation rate and retention rate.

The modeling method we employ is as follows: first of all, we use the correlation analysis and cluster analysis to screen out six evaluation indicators, which are selected with the consideration of avoiding the focus of other large grant organizations.

Then, we take advantage of the Q-value based method and establish a mathematical model based on the Markowitz Portfolio Theory model to give a 1 to N optimized and prioritized candidate list of schools. With regard to the time duration after which the donations start to work, we introduce the historical investment factor to the ROI calculation formula. And the years when the rankings of the universities vary a lot from the first year's is defined as the required time.

The major result of our optimal investment strategy is as follows, while the time duration is around 7 years. (To save space, we only list the top five universities)

Table .Rank of the investment on universities (min_investment=500,000)

Rank	University	Investment amount	ROI
1	***TAMU***	**15500000**	**15.109**
2	***UCB***	**8000000**	**15.0923**
3	***UCLA***	**7000000**	**15.0901**
4	***UM***	**7000000**	**15.1**
5	***UFL***	**6500000**	**15.0967**

It is obvious that the top five universities enjoy a high reputation, which means that investing these universities will have the highest likelihood of improving the performance of the undergraduates.

One of the advantages of our model is that we make full use of all kinds of data of the IPEDS database, including the data of the last six years. In this case we make a full assessment of the situations of the universities and find out as many effective indicators as possible to quantify the benefit of the donation meanwhile.

Another advantage of our model is universally applicable. Due to the fact that we introduce the historical investment factor in the ROI calculation formula, we can apply the model across both long term and short term investment. Therefore, our model has high practical value.

We hope that our model and candidate list of schools we proposed can be of remarkably use to you.

Sincerely yours,
Team 49652